Meeting: 1004, Bowling Green, Kentucky, SS 6A, Special Session on Representation Theory

1004-22-194 William Graham* (wag@math.uga.edu), University of Georgia, Department of Mathematics, Boyd Graduate Studies Research Center, Athens, GA 30602, and Markus Hunziker (Markus_Hunziker@baylor.edu), Department of Mathematics, Baylor University, Waco, TX 76798. Multiplication of polynomials on Hermitian symmetric spaces and Littlewood-Richardson coefficients. Preliminary report.
Let $K$ be a complex reductive algebraic group and $V$ a representation of $K$. Let $S$ denote the ring of polynomials on $V$. Assume that the action of $K$ on $S$ is multiplicity-free. If $\lambda$ denotes the isomorphism class of an irreducible representation of $K$, let $\rho_{\lambda}: K \rightarrow G L\left(V_{\lambda}\right)$ denote the corresponding irreducible representation, and $S_{\lambda}$ the $\lambda$-isotypic component of $S$. Write $S_{\lambda} \cdot S_{\mu}$ for the subspace of $S$ spanned by products of $S_{\lambda}$ and $S_{\mu}$. If $V_{\nu}$ occurs as an irreducible constituent of $V_{\lambda} \otimes V_{\mu}$, is it true that $S_{\nu} \subset S_{\lambda} \cdot S_{\mu}$ ? We study this question for representations arising in the context of Hermitian symmetric pairs. We prove that the answer is yes in some cases, and using results of Ruitenberg, we show that in the remaining classical cases, the answer is yes provided that a conjecture of Stanley on the multiplication of Jack polynomials is true. We also show how the conjecture connects multiplication in the ring $S$ to the usual Littlewood-Richardson rule. (Received January 25, 2005)

