Meeting: 1006, Lubbock, Texas, SS 5A, Special Session on Recent Advances in Complex Function Theory

1006-32-211 John A. Pfaltzgraff (jap@math.unc.edu), Department of Mathematics, University of North Carolina, Chapel Hill, NC 25799, and Ted J. Suffridge* (ted@ms.uky.edu), Department of Mathematics, University of Kentucky, Lexington, KY 40506. Linear Invariant Families on the Ball, Extremal Problems and Invariant Mappings. Preliminary report.

A linear invariant family \mathcal{F} on the unit ball $B \subset \mathbb{C}^n$ is a family of holomorphic mappings $f : B \to \mathbb{C}^n$, that are locally univalent, normalized by requiring f(0) = 0, Df(0) = I and having the property $K_{\phi}(f) \in \mathcal{F}$ whenever $f \in \mathcal{F}$. Here, given a holomorphic automorphism ϕ of the ball, $K_{\phi}(f)$ is the Koebe transform $D\phi(0)^{-1}Df(\phi(0))^{-1}(f(\phi(z)) - f(\phi(0)))$, i.e. the composition of f with ϕ renormalized. The norm order of a linear invariant family $\|ord\|\mathcal{F}$ is given by $\sup_{f \in \mathcal{F}} \{\|D^2f(0)\|\}$. As an example, the family of normalized holomorphic mappings of the ball onto convex domains has norm order 1. We discuss the connection between the problem of finding $\max_{f \in \mathcal{F}} \operatorname{Re}(J(f))$ for a linear functional J on \mathcal{F} and invariant functions, i.e. functions that have the property $K_{\phi}(f) = f$, for a family of automorphisms, ϕ . (Received February 15, 2005)