Robert M. Sulman* (rsulman@centralmethodist.edu), Central Methodist University, Fayette, MO 65248. Disruption of Symmetry Creates New Symmetries. Preliminary report.
The quadratic $\mathrm{f}(\mathrm{x})=\mathrm{ax}^{2}+\mathrm{bx}+\mathrm{c}(\mathrm{a}>0)$ has symmetry about a vertical line. When we divide f by $\mathrm{g}(\mathrm{x})=\mathrm{px} \mathrm{x}^{2}+\mathrm{qx}+\mathrm{r}(\mathrm{p}>0)$ the symmetry above is disrupted. However, new symmetries are created and they are examined in this talk. Specifically, the graph of $h=f / g$ will always have a minimum value $y=m$ and a maximum value $y=M$ when $\operatorname{disc}(g)$ is negative and the axes of symmetry of f and g are different. If this is the case, then: (i) $\operatorname{Mm}=\operatorname{disc}(\mathrm{f}) / \operatorname{disc}(\mathrm{g})(\mathrm{ii}) M m=\beta(\mathrm{f}) / \min (\mathrm{g})$, where $\beta$ is the horizontal asymptote of $h$ and $y=\min (f), y=\min (g)$ are the minimum values of $f$ and $g$ (occurring at each vertex). As a consequence of (ii) we see that if either graph(f) or $\operatorname{graph}(\mathrm{g})$ are translated horizontally (not to share axis of symmetry), the resulting rational function will have a minimum and maximum value whose product is Mm . That is, the product of extreme values is invariant under horizontal shifts of the numerator and denominator. A look at what this means geometrically and some concrete examples are given. This result is derived solely using methods of Pre-Calculus and is thus accessible to anyone with such background. (Received September 21, 2009)

