On Graham's Tree Reconstruction Conjecture.
Suppose $G$ is a tree. If we are given only the integer sequence $\left(|V|,\left|V^{1}\right|,\left|V^{2}\right|, \ldots\right)$ where $V^{k}=V\left(L^{k}(G)\right)$ is the $k^{\text {th }}$ iterated line graph's vertex set, is it possible to determine the original tree? This question ("Graham's Tree Reconstruction Conjecture") has only been answered for very limited classes of graphs. First, we show that certain obvious counterexamples cannot exist. Then we proceed to use the theory of partitions to bound the number of steps it could take to determine the original tree. In this paper, we are able to show that the trees break into quadratically many equivalence classes after the first line graph. Next, we study a closely related problem: Define $A_{G}$, the adjacency matrix of a graph $G$ on $n$ vertices, as the $n \times n$ matrix $\left\{a_{i, j}\right\}_{i, j=1}^{n}$ with $a_{i j}=1$ if $\{i, j\} \in E(G), 0$ otherwise. Denote by $W_{k}$ the quantity $1^{\top} A_{G}^{k} \mathbf{1}$, where $\mathbf{1}$ is the all 1 's vector, i.e., the number of walks of length $k$ in $G$. Call the sequence $\left\{W_{k}\right\}_{k=0}^{\infty}$ the walk sequence of a graph. We apply ideas from spectral and fractional graph theory to obtain results about the walk sequences and their connection to the "Graham's Tree Reconstruction Conjecture." (Received August 29, 2009)

