1056-14-41 Joe Harris* (harris@math.harvard.edu), 1 Oxford St., Cambridge, MA 02138. The Interpolation Problem.
A fundamental theorem in algebra says that, given two sets of complex numbers $z_{0}, \ldots, z_{d}$ and $a_{0}, \ldots, a_{d}$, there exists a unique polynomial $f(z) \in \mathbb{C}[z]$ of degree at most $d$ such that $f\left(z_{i}\right)=a_{i}$ for all $i$.

This is a beautiful and highly useful result. But when we ask the natural next question-what can we say about polynomials in several variables-we enter a realm of mystery. The analogous statement for polynomials in two or more variables is visibly false, but no one knows exactly when, and by how much, it can fail. This gives rise to a whole class of problems, collectively known as interpolation problems.

The interpolation problem is like a number of problems in algebraic geometry: it's completely elementary to state; a general solution seems beyond us; and yet substantial progress has been made and is currently being made. In this talk I'll try to give an elementary introduction to the problem and what we know about it. In particular, I'll try to describe a common thread in the known and conjectured solutions of special cases, giving a geometric characterization of when interpolation fails in general. (Received September 22, 2009)

