Srinivasan Balaji* (balaji@gwu.edu), Department of Statistics, 2140 Pennsylvania Avenue NW, Washington, DC 20052, and Hosam M Mahmoud (hosam@gwu.edu), Department of Statistics, 2140 Pennsylvania Avenue NW, Washington, DC 20052. Phases in the Mixing of Gases via the Ehrenfest Urn Model.
The Ehrenfest urn is a model for the mixing of gases in two chambers. Classic research deals with this system as a Markovian model with a fixed number of balls, and derives the steady-state behavior as a binomial distribution (which can be approximated by a normal distribution). We study the gradual change for an urn containing $n$ balls from the initial condition to the steady state. We look at the status of the urn after $k_{n}$ draws. We identify three phases of $k_{n}$ : The growing sublinear, the linear, and the superlinear. In the growing sublinear phase the amount of gas in either chamber is normally distributed, with parameters that are influenced by the initial conditions. In the linear phase a different normal distribution applies, in which the influence of the initial conditions is attenuated. The steady state is not a good approximation until a superlinear amount of time has elapsed. At the superlinear stage the mix is nearly perfect, with a nearly perfect symmetrical normal distribution in which the effect of the initial conditions is completely washed away. We give interpretations for how the results in different phases conjoin at the "seam lines." The Gaussian results are obtained via martingale theory. (Received September 22, 2009)

