1056-91-1642    **John Cloutier**, Department of Mathematics, University of California at Santa Barbara, Santa Barbara, CA 93106, **Kathryn L. Nyman\*** (`knyman@willamette.edu`), Department of Mathematics, Willamette University, Salem, OR 97301, and **Francis Edward Su**, Department of Mathematics, Harvey Mudd College, Claremont, CA 91711. *Two player envy-free multi-cake division using a polytopal Sperner's lemma.*

We introduce a generalized cake-cutting problem in which we seek to divide multiple cakes so that two players may get their most-preferred piece selections: a choice of one piece from each cake, allowing for the possibility of linked preferences over the cakes. For two players, we show that disjoint envy-free piece selections may not exist for two cakes cut into two pieces each, and they may not exist for three cakes cut into three pieces each. However, there do exist such divisions for two cakes cut into three pieces each, and for three cakes cut into four pieces each. The resulting allocations of pieces to players are Pareto-optimal with respect to the division. We use a generalization of Sperner's lemma on the polytope of divisions to locate solutions to our generalized cake-cutting problem. (Received September 22, 2009)