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Ira M. Gessel* (gessel@brandeis.edu), Department of Mathematics, MS 050, Brandeis University, Waltham, MA 02453. *Combinatorial Proofs of Congruences*.

In 1872, Julius Petersen published a frequently rediscovered combinatorial proof of Fermat's theorem $a^p \equiv a \pmod{p}$, where p is a prime: If we color the spokes of a p -spoke wheel in a colors, and call two colorings equivalent if one can be rotated into the other, then every equivalence class contains p colorings except for the a equivalence classes consisting of a single monochromatic coloring. Petersen gave a similar proof of Wilson's theorem $(p-1)! \equiv -1 \pmod{p}$, and Lucas's theorem $\binom{ap+b}{cp+d} \equiv \binom{a}{c} \binom{b}{d} \pmod{p}$, where $0 \leq b, d < p$ can be proved by the same idea: if a group of order p (or a power of p) acts on finite set S then the size of every orbit is either 1 or a multiple of p , so $|S|$ is congruent modulo p to the number of fixed points.

I will describe how this approach can be applied to find congruences for Bell numbers, derangement numbers, and other sequences of combinatorial interest. (Received September 15, 2009)