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Urbana-Champaign, 1409 W Green Street, Urbana, IL 61801, and **Xuding Zhu**. *Decomposition of sparse graphs using forests and a graph with bounded degree.*

From the Matroid Union Theorem by Edmonds and Nash-Williams, the Tree Packing Theorem will immediately follow: a graph decomposes into k forests if and only if the arboricity $\max_{H \subseteq G} \lceil |E(H)| / (|V(H)| - 1) \rceil$ is at most k . We consider the *fractional arboricity* $\max_{H \subseteq G} \frac{|E(H)|}{|V(H)| - 1}$. The Nine Dragon Tree (NDT) Conjecture, posted by Montassier et al., states that if the fractional arboricity of a graph is at most $k + \frac{d}{k+d+1}$, then the graph decomposes into $k + 1$ forests, with one of them having maximum degree at most d .

For $d \geq k + 1$, we prove a sharp sparseness condition for decomposability into k forests and a graph having maximum degree at most d . Consequences are that every graph with fractional arboricity at most $k + d/(k + d + 1)$ has such a decomposition. For $d \leq k + 1$, we prove that every graph with fractional arboricity at most $k + d/(2k + 2)$ decomposes into $k + 1$ forests, with one of them having maximum degree at most d . This implies the NDT Conjecture for the case $d = k + 1$. Also, for $k = 1$, we prove that the NDT Conjecture is true for $d \leq 6$. (Received September 19, 2010)