1067-05-688 **David L. Clampitt*** (`clampitt.4@osu.edu`). *The Coin Problem, Central Words, and Guido of Arezzo.*

The classical coin problem asks which integer values $n \geq 0$ are representable with coins of denominations p and q, where $\mathrm{gcf}(p,q) = 1$. The greatest integer not representable (the Frobenius number) is $pq-(p+q)$, so the $p+q-1$ integers from $(p-1)(q-1)$ to $pq-1$ are representable. In a recent paper, Paquin and Reutenauer revisit the coin problem in relation to Christoffel words. The Christoffel word of length $p+q$ on the alphabet $A = \{a,b\}$ has the structure aub, where u is a central word, the unique palindrome with periods p and q (i.e., if $u = u_1 \cdots u_{p+q-2}$, then $u_k = u_{k+p}$ for $1 \leq k < q-2$ and $u_k = u_{k+q}$ for $1 \leq k < p-2$). In this paper, it is shown that u is encoded by the $p+q-2$ differences in $Zp \times Zq$ between solutions to adjacent values n and $n+1$, $pq-(p+q) < n < pq-1$. Central words also encode constructions by the 11th-century music theorist, Guido of Arezzo: his hexachord, and the diamond diagram from his treatise, Micrologus. In these cases, the values p and q represent the generic lengths (spans) of perfect fourths (3) and fifths (4), and perfect fifths (4) and octaves (7), respectively. The conjunction of these facts is asserted to be productive on the music-theoretical side. (Received September 13, 2010)