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Bobbe J. Cooper and **Eric S. Rowland*** (erowland@tulane.edu), Mathematics Department, Tulane University, New Orleans, LA 70118, and **Doron Zeilberger**. *Toward a language theoretic proof of the four color theorem.*

Consider the simple context-free grammar G consisting of start symbols $0, 1, 2$ and formation rules $0 \rightarrow 12, 0 \rightarrow 21, 1 \rightarrow 02, 1 \rightarrow 20, 2 \rightarrow 01, 2 \rightarrow 10$. Observe that G is ambiguous — there exist distinct trees that parse the same word. For example, the trees $((()())())$ and $((()())())$ both parse the word 010 .

However, something much stronger can be said about this grammar.

Theorem 1 *The grammar G is totally ambiguous.*

That is, for every pair of n -leaf derivation trees, there exists a length- n word on $\{0, 1, 2\}$ that both trees parse. In 1990 Louis Kauffman proved this theorem by showing that it is equivalent to the four color theorem.

Here we take the opposite approach, the hope being to prove Theorem 1 directly and thereby obtain a shorter proof of the four color theorem as well as additional insight. In this direction we enumerate the common parse words for several infinite families of tree pairs and discuss ways to reduce the problem of finding a parse word for a pair of trees to that for a smaller pair. (Received July 20, 2010)