1067-05-872 **Fan Wei*** (`fan_wei@mit.edu`), 6-301, 471 Memorial Drive, Cambridge, MA 02139. *The Weak Bruhat Order and Separable Permutations.*

Let $\mathfrak{S}_n$ denote the symmetric group of all permutations of $1, 2, \ldots n$, partially ordered by the weak Bruhat order. Thus for any permutation $w \in \mathfrak{S}_n$, the rank $\ell(w)$ of w is the number of inversions in w. It follows that the rank-generating function of $\mathfrak{S}_n$ is

$$F(\mathfrak{S}_n, q) = \sum_{w \in \mathfrak{S}_n} q^{\ell(w)} = (n)!,$$

where $(n) = (1)(2)\cdots(n)$ and $(i) = 1 + q + q^2 + \cdots + q^{i-1}$.

For any $w \in \mathfrak{S}_n$, we define two graded posets associated with w: $\Lambda_w = \{v \in \mathfrak{S}_n \colon v \leq w\}$ and $V_w = \{v \in \mathfrak{S}_n \colon v \geq w\}$. In V_w, we define the rank of v to be $\ell(v) - \ell(w)$. We will show that if w is separable (i.e., 3142 and 2413-avoiding), then there is the surprising formula

$$F(\Lambda_w, q)F(V_w, q) = (n)!.$$

Moreover, we define a bijection $\varphi\colon \Lambda_w \times V_w \to \mathfrak{S}_n$ satisfying $\ell(u) + \ell(v) - \ell(w) = \ell(\varphi(u, v))$, and we give an explicit formula for $F(\Lambda_w, q)$ and $F(V_w, q)$. We also show that Λ_w and V_w are rank-symmetric and rank-unimodal for any separable permutation w.

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