Joseph H Silverman* (jhs@math.brown.edu), Mathematics Department - Box 1917, Brown University, Providence, RI 02912. Lehmer's conjecture and points on elliptic curves that are congruent to torsion points. Preliminary report.
Lehmer's conjecture says that if $\alpha \in \overline{\mathbb{Q}}$ is not a root of unity, then its height satisfies $h(\alpha) \geq C / D(\alpha)$, where $C$ is an absolute constant and $D(\alpha)=[\mathbb{Q}(\alpha): \mathbb{Q}]$. In addition to being of theoretical interest, height estimates of this sort are used to determine termination conditions for various search algorithms. Borwein, Dobrowolski and Mossinghoff have shown that if the minimal polynomial $F_{\alpha}(X)$ of $\alpha$ is congruent modulo $m$ to $1+X+X^{2}+\cdots+X^{D(\alpha)}$, then the height of $\alpha$ satisfies a Lehmer bound of the form $h(\alpha) \geq(C \log m) / D(\alpha)$. Interpreting the congruence as saying that the conjugates of $\alpha$ are congruent to roots of unity, we formulate and prove an elliptic curve analogue of the $[\mathrm{BDM}]$ result. (Received July 24, 2010)

