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Maxwell Anselm* (mba210@lehigh.edu) and Steven H Weintraub (shw2@lehigh.edu). *A Generalization of Continued Fractions.*

We consider a generalization of continued fractions, where the 1 in the “numerator” of the continued fraction is replaced by an arbitrary positive integer N . We refer to this as a cf_N expansion. The algorithm for cf_1 expansions (i.e., classical continued fractions) generalizes to cf_N expansions. Also, there is a natural notion of a *best* cf_N expansion.

Theorem 1. *For $N > 1$, every positive irrational number has infinitely many cf_N expansions and infinitely many of these expansions are nonperiodic. For $N > 1$, every positive rational number has infinitely many finite cf_N expansions. For $N > 2$, every positive rational number has infinitely many periodic cf_N expansions and infinitely many nonperiodic cf_N expansions.*

Conjecture 2. *For $N > 1$, the best cf_N expansion of a quadratic irrationality is not always periodic.*

We also consider the relationship between the best cf_N expansion of \sqrt{E} and solutions to Pell’s equation $x^2 - Ey^2 = 1$, where we find that the relationship is considerably more involved than in the classical case.

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