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Fix a positive integer  $\ell$ , and let  $K$  be any field containing  $\zeta_\ell + \zeta_\ell^{-1}$  but not  $\zeta_\ell$ . Rikuna discovered a polynomial  $F_\ell$  over the function field  $K(T)$  whose Galois group is  $\mathbb{Z}/\ell\mathbb{Z}$ . Komatsu recently generalized classical Kummer theory to cover cyclic extensions arising from  $F_\ell$ .

In our work, for each  $m \geq 1$ , we introduce the  $m$ -th *generalized Rikuna polynomial*  $r_m$ , which roughly is formed from the  $m$ -th iteration of a rational function related to  $F_\ell$ . Let  $K_m$  be the splitting field of  $r_m$  over  $K(T)$ . It is known that the tower of  $K_m$ 's ramifies at finitely many primes of  $K(T)$ .

We study the tower of  $K_m$ 's. For any odd  $\ell \geq 3$ , we show that the Galois group  $\text{Gal}(K_m/K(T))$  is a semi-direct product  $\mathbb{Z}/\ell^m\mathbb{Z} \rtimes \mathbb{Z}/(\ell^m/b_m)\mathbb{Z}$ , where  $b_m$  is the order of a certain group of roots of unity in  $K_m$ . For even  $\ell$ , the Galois group is one of four possibilities, depending on the field  $K$ . We also show that only one prime of  $K(T)$  ramifies in the tower of  $K_m$ 's, and determine this prime explicitly. Then, using the Riemann-Hurwitz formula, we prove that  $K_m$  is of genus 0, and therefore has class number 1, for all  $m \geq 1$ . (Received September 19, 2010)