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**Kiumars Kaveh\*** ([kaveh@pitt.edu](mailto:kaveh@pitt.edu)), 301 Thackeray Hall, Department of Mathematics,  
University of Pittsburgh, Pittsburgh, PA 15260. *Newton-Okounkov bodies and crystal bases.*

Let  $G$  be a connected reductive algebraic group. We prove that the string parametrization of a crystal basis for a finite dimensional irreducible representation of  $G$  coincides with a natural valuation, on the field of rational functions on the flag variety  $G/B$ , constructed out of a coordinate system on a Bott-Samelson variety (or a flag of Schubert varieties on  $G/B$ ). This shows that the string polytopes associated to irreducible representations, can be realized as Newton-Okounkov bodies for the flag variety. This fully generalizes an earlier result of A. Okounkov for Gelfand-Cetlin polytopes of symplectic group. As another corollary we deduce a multiplicativity property of the canonical basis due to Caldero. We generalize the results to spherical varieties. From these existence of SAGBI bases for homogeneous coordinate rings of flag and spherical varieties, as well as their toric degenerations follow (recovering toric degenerations of Alexeev-Brion, Caldero and the speaker). (Received September 20, 2010)