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Andrew J. Bernoff* (ajb@hmc.edu), Department of Mathematics, Harvey Mudd College, Claremont, CA 91711, and Chad M. Topaz (ctopaz@macalester.edu), Dept. of Mathematics, Statistics, and CS, Macalester College, St. Paul, MN 55105. A Primer of Swarm Equilibria.

We study equilibrium configurations of swarming biological organisms subject to exogenous and pairwise endogenous forces. Beginning with a discrete dynamical model, we derive a variational description of the continuum population density. Equilibrium solutions are extrema of an energy functional, and satisfy a Fredholm integral equation. We find conditions for the extrema to be local minimizers, global minimizers, and minimizers with respect to infinitesimal Lagrangian displacements of mass. In one spatial dimension, for a variety of exogenous forces, endogenous forces, and domain configurations, we find exact analytical expressions for the equilibria. These agree closely with numerical simulations of the underlying discrete model. The exact solutions provide a sampling of the wide variety of equilibrium configurations possible within our general swarm modeling framework. The equilibria typically are compactly supported and may contain δ -concentrations or jump discontinuities at the edge of the support. We apply our methods to a model of locust swarms, which are observed in nature to consist of a concentrated population on the ground separated from an airborne group. Our model can reproduce this configuration; quasi-two-dimensionality of the model plays a critical role. (Received August 20, 2010)