In this paper, we investigate a particular Diophantine equation, $X^{2}+Y^{3}=6912 Z^{2}$, and a set of solutions to the equation, which are derived from some polynomials in $\mathbf{Z}[x, y]$. We focus on three polynomials $X=f(x, y), Y=g(x, y)$ and $Z=h(x, y)$ that satisfy the Diophantine equation and the greatest common divisors for the the integer values of the polynomials. These polynomials are relatively prime in $\mathbf{Q}[x, y]$. However, for a fixed integer pair $x_{0}, y_{0}$, the integer values $f\left(x_{0}, y_{0}\right), g\left(x_{0}, y_{0}\right)$ and $h\left(x_{0}, y_{0}\right)$ are not necessarily relatively prime in $\mathbf{Z}[x, y]$. We investigate the greatest common divisors (GCDs) between these three polynomial values for specific integer pairs $x_{0}$ and $y_{0}$. We focus on the cases where $y=1$ and $y=2$. For these cases, we give complete classifications on the distribution of the GCDs. We use the Gröbner Bases technique as an aid in investigating the GCDs for $f, g, h$ in $\mathbf{Z}[x, y]$. We then generalize the results from the cases $y=1$ and $y=2$ to obtain similar properties for the GCDs of $f, g, h$ for all $x$ and $y$ in $\mathbf{Z}[x, y]$. (Received September 17, 2010)

