Thomas J Dinitz* (tdinitz@students.colgate.edu), 152 Forest's Edge, Hinesburg, VT 05461, and Matthew Hartman and Jenya Soprunova (soprunova@math.kent.edu). Tropical determinants and cheating when solving the Rubik's cube.
Consider the usual Rubik's cube with 9 square stickers on a side, and each sticker colored in one of six colors. Instead of solving it in the normal way (by rotating faces), we want to solve it by peeling off and replacing the stickers. In this presentation we will address the natural question: What is the maximum number of stickers you would ever need to peel off and replace? We first show that this problem can be translated to the language of matrices. Let A be a $6 \times 6$ matrix whose $(i, j)$ th entry is the number of squares of color $i$ in face $j$ of the cube. Note that the matrix A has all its row and column sums equal to 9 . The number of stickers that we do not need to peel off and replace is equal to the tropical determinant of A. We would like to find a matrix A with the lowest possible tropical determinant as this would provide an example of a Rubik's cube which needs the most number of stickers replaced. Stated more formally, our initial problem reduces to finding a sharp lower bound on the tropical determinant of $6 \times 6$ integer matrices A with non-negative entries and row and column sums equal to 9 . We solve this problem as well as the general problem of $n$-by- $n$ matrices with row and column sums equal to $m$. (Received September 12, 2010)

