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Teaching Basic Number Theory from the Sieve of Eratosthenes.

Given a natural number n , the Sieve of Eratosthenes is an algorithm for finding all prime numbers between 1 and n . I teach a Liberal Arts mathematics course, Mathematical Thinking, specifically targeted at the “I can’t do math” student. The first homework includes using the Sieve of Eratosthenes to find all primes between 1 and 200. This pedestrian assignment is actually a foundation stone for the rest of the course. From here we prove: 1. What does it mean to be a multiple of 3 or 5 or 7? 2. Could 1 more than a multiple of 7, that is $7k+1$, ever be a multiple of 7? 3. What are the possible remainders when one divides by 7? 4. Extend the “Even/Odd” partition notion of multiples of 2 to other multiples. Think in terms of remainders. Using patterns observed in the Sieve of Eratosthenes we are able to show that: There are an infinite number of primes (#2). An odd times odd is always odd (#2). Any rational number m/n can be written as a repeating decimal (#3). The square root of 3 is irrational (#1). Hilbert’s Hotel can accommodate three infinitely full busses (#4). We can do modular arithmetic (#4).

The Sieve of Eratosthenes does motivate much of a Mathematical Thinking Course! (Received September 20, 2010)