Parametric cubic polynomial curves are useful in applications, being of relatively low dimension, and yet, flexible in their shape. To use these curves fully, one must completely understand the cases of singularity, i.e., where the speed of the parametric curves is zero.

Parametric polynomial tensor product surfaces are useful as well, and these are often most easily generated from a network of curves. For these surfaces, singularity occurs when the normal vector, defined as the cross product of the two partial derivatives, is zero.

These singularities are described here in terms of the Bézier form, a representation of parametric polynomial curves and surfaces employing the Bernstein polynomials as basis functions, in which the coefficients have geometric significance. Bézier curves and surfaces, which are used extensively in computer graphics, computer-aided design, and related fields, were first developed in the 1950s and 60s in the French automobile industry.

An industry application motivating the present work comes from computer-aided design of ship hulls. (Received September 22, 2010)

