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**Luboš Pick\*** ([pick@karlin.mff.cuni.cz](mailto:pick@karlin.mff.cuni.cz)), Faculty of Mathematics and Physics, KMA, Charles University, Sokolovská 83, 186 75 Prague, Czech Rep. *Sobolev embeddings - old and new.*

We will give a survey of results obtained recently in a series of papers written jointly with Ron Kerman. The main aim is to study Sobolev-type imbedding inequalities involving rearrangement-invariant Banach function norms. We are in particular interested in the question when such norms are optimal for the embedding. As a first step we establish the equivalence of a Sobolev imbedding to the boundedness of a certain weighted Hardy operator. This Hardy operator is then used to prove the existence of rearrangement-invariant norms that are optimal in the imbedding inequality. Our approach is to use the methods and principles of Interpolation Theory. We then give explicit expressions for the optimal partners corresponding to a given domain or range norm.

Finally, we find necessary and sufficient conditions in order that a Sobolev space be compactly imbedded into a rearrangement-invariant space. We establish the equivalence of the compactness of the Sobolev imbedding with the compactness of the above-mentioned Hardy operator. We illustrate the results with examples involving Orlicz and classical Lorentz norms. (Received February 06, 2006)