1019-42-199 **Leonid Slavin*** (`slavin@math.uconn.edu`), Department of Mathematics, University of Connecticut, 196 Auditorium Road, U-3009, Storrs, CT 06269, and **Vasily Vasyunin** (`vasyunin@pdmi.ras.ru`), St. Petersburg Department, Steklov Mathematical Institute, RAS, 27 Fontanka, St. Petersburg, 191023, Russia. *Sharp constants in the integral John-Nirenberg inequality and the corresponding weak-form estimates.*

On an interval I, we study the integral form of the John-Nirenberg inequality for BMO(I) (with the L^2-based BMO norm). Two different versions of BMO are considered, the continuous and dyadic one. We apply the Bellman function method to the problem, explicitly find the Bellman function and thus prove the following

Theorem. *There exists $\varepsilon_0 > 0$ such that for every $0 \le \varepsilon < \varepsilon_0$ there is $C(\varepsilon) > 0$ such that for any function $\varphi \in \mathrm{BMO}_\varepsilon(I)$,*

$$\langle e^\varphi \rangle_I \le C(\varepsilon) e^{\langle \varphi \rangle_I}.$$

Here $\mathrm{BMO}_\varepsilon(I)$ is the ε-ball in BMO(I). The sharp continuous-case results are $\varepsilon_0 = 1$ and $C(\varepsilon) = \frac{e^{-\varepsilon}}{1-\varepsilon}$. The dyadic results are significantly different. In particular, $\varepsilon_0^d = \sqrt{2}\log 2$.

This integral inequality immediately produces the weak-form estimate

$$\frac{1}{|I|}|\{\varphi - \langle \varphi \rangle_I > \lambda\}| \le \left(1 + \frac{\lambda}{\|\varphi\|}\right) e^{-\lambda/\|\varphi\|}$$

for any $\varphi \in \mathrm{BMO}(I)$. Here $\langle \varphi \rangle_I = \frac{1}{|I|}\int_I \varphi$. This estimate is better (for large λ) than the classical estimate $C_1 e^{-c_2\lambda/\|\varphi\|}$ for any $c_2 < 1$. (Received August 15, 2006)