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Vasyl Dmytrenko, Felix Lazebnik and Jason Williford* (jsw@wpi.edu), Department of Mathematical Sciences, Worcester Polytechnic Institute, 100 Institute Road, Worcester, MA 01609-2280. *Monomial Graphs of Girth at Least Eight.*

Let $q = p^e$ where e is a positive integer and p an odd prime, and let \mathbb{F}_q be the finite field of q elements. Let $f_2, f_3 \in \mathbb{F}_q[x, y]$ be monomials. A monomial graph $G = G_q(f_2, f_3)$ is a bipartite graph with vertex partitions $P = \mathbb{F}_q^3$ and $L = \mathbb{F}_q^3$, and edges defined as follows: a vertex $(p) = (p_1, p_2, p_3) \in P$ is adjacent to a vertex $[l] = [l_1, l_2, l_3]$ if and only if $p_2 + l_2 = f_2(p_1, l_1)$ and $p_3 + l_3 = f_3(p_1, l_1)$.

Motivated by some questions in finite geometries and extremal graph theory, we ask when G has girth at least eight. We show that for $q \geq 5$ and odd, and $e = 2^a 3^b$, a monomial graph of girth at least eight is isomorphic to the graph $G_q(xy, xy^2)$, which is an induced subgraph of the classical generalized quadrangle $W(q)$. For all other e , we show that a monomial graph is isomorphic to a graph $G_q(xy, x^k y^{2k})$, with $1 \leq k \leq (q-1)/2$ and satisfying several other strong conditions. These conditions imply that $k = 1$ for all $q < 10^{10}$. In particular, for a given positive integer k , graph $G_q(xy, x^k y^{2k})$ can be of girth eight only for finitely many odd characteristics p . (Received February 27, 2007)