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Yufang Hao*, Department of Applied Mathematics, University of Waterloo, 200 University Ave. West, Waterloo, Ontario N2L3G1. *A Generalization of the Theorem of Whittaker-Shannon-Kotelnikov*. Preliminary report.

We will discuss a generalization of the theorem of Whittaker-Shannon-Kotelnikov (WSK) by using the theory of self-adjoint extensions of a simple symmetric operator. The WSK theorem states that any continuous function $f(t)$ in the space of B -bandlimited functions can be reconstructed from its values on a discrete set of points $\{t_n\}_n$ via $f(t) = \sum_n G(t, t_n)f(t_n)$, where the sampling points are equidistant $t_{n+1} - t_n = 1/(2B)$, and reconstruction kernel $G(t, t_n)$ is $\text{sinc}(2B(t - t_n))$. By observing that the Fourier transforms of such B -bandlimited functions are invariant under the derivative operator $i\frac{d}{dt}$, we interpret the sampling function space as the invariant domain of the multiplication operator T where $Tf(t) = tf(t)$. The operator T is a simple symmetric operator with deficiency indices $(1, 1)$. In the special case of the WSK theorem, all the self-adjoint extensions of T have equidistant eigenvalues. Hence the WSK theorem is restricted to uniform sampling. By considering a general symmetric operator T , we can generalize the WSK theorem to allow non-equidistant sampling points. The new function space is the domain of T , and an explicit expression of the generalized reconstruction kernel is obtained. (Received February 12, 2008)