1077-35-1632 Gigliola Staffilani* (gigliola@math.mit.edu), Room 2-246, 77 Massachusetts Avenue, Cambridge, MA 02139, and Kay Kirkpatrick and Enno Lenzmann. On the continuum limit for discrete NLS with long-range lattice interactions.

We consider a general class of discrete nonlinear Schrödinger equations (DNLS) on the lattice $h\mathbb{Z}$ with mesh size h > 0. In the continuum limit when $h \to 0$, we prove that the limiting dynamics are given by a nonlinear Schrödinger equation (NLS) on \mathbb{R} with the fractional Laplacian $(-\Delta)^{\alpha}$ as dispersive symbol. In particular, we obtain that fractional powers $\frac{1}{2} < \alpha < 1$ arise from long-range lattice interactions when passing to the continuum limit, whereas NLS with the non-fractional Laplacian $-\Delta$ describes the dispersion in the continuum limit for short-range lattice interactions (e.g., nearest-neighbor interactions).

Our results rigorously justify certain NLS model equations with fractional Laplacians proposed in the physics literature. Moreover, the arguments given in our paper can be also applied to discuss the continuum limit for other lattice systems with long-range interactions. (Received September 20, 2011)