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Gregory Berkolaiko and Peter Kuchment* (kuchment@math.tamu.edu), Mathematics Department, Texas A&M University, College Station, TX 77843-3368, and Uzy Smilansky. A nodal domain count mystery.

In this talk we address the nodal count (i.e., the number of sub-domains where the function preserves its sign) for eigenfunctions of Laplace or Schroedinger operators with Dirichlet boundary conditions in bounded domains (billiards). The classical Sturm theorem claims that in dimension one, the nodal and eigenfunction counts coincide: the n-th eigenfunction has exactly n nodal domains. The Courant Nodal Theorem claims that in any dimension, the number of nodal domains of the n-th eigenfunction cannot exceed n. However, it is known that in dimensions higher than 1 the equality may hold for only finitely many eigenfunctions. Thus, in most cases a "nodal deficiency" arises. Moreover, examples are known of eigenfunctions with an arbitrarily large index n that have just two nodal domains. One can say there is essentially no understanding of the nodal deficiency.

We show that, under some conditions, the nodal deficiencies coincide with the Morse indices of critical points of some functional. (Received September 15, 2011)