Lillian B. Pierce* (lbpierce@princeton.edu), Department of Mathematics, Fine Hall, Princeton University, Princeton, NJ 08544. Fractional integration on the Heisenberg group: A discrete analogue.

A discrete analogue of fractional integration on the Heisenberg group may be defined by

$$T^{\lambda}f(n,t) = \sum_{m \in \mathbb{Z}^{2k}, m \neq 0} \frac{f(n-m, t-\langle n, m \rangle)}{|m|^{2k\lambda}},$$

where f is a function of $\mathbb{Z}^{2k} \times \mathbb{Z}$, $0 < \lambda < 1$, $n = (n_1, n_2) \in \mathbb{Z}^k \times \mathbb{Z}^k$ denotes the element $n_1 + in_2 \in \mathbb{C}^k$, and $\langle n, m \rangle = 2\Im(n \cdot \bar{m})$ denotes the symplectic bilinear form on the Heisenberg group. The main question is for which p, q, λ does T^{λ} map $\ell^p(\mathbb{Z}^{2k+1})$ to $\ell^q(\mathbb{Z}^{2k+1})$? This work proves results of this type for a wide range of p, q, λ , and furthermore considers the case where $\langle \cdot, \cdot \rangle$ is replaced by more general symplectic bilinear forms. While these are analytic results, key aspects of the method come from number theory, and in particular, the circle method. (Received August 10, 2008)