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Euler's theorem and the "Plaited Mat" Sona Designs of Africa.

The simplest form of the sona designs of the Chokwe (or Cokwe) people of Angola/Congo can be viewed as mirror curves in a rectangular grid, with lines passing through the rectangle at 45° angles and bouncing off the walls to form a closed curve. A well-known result is that the number of lines needed to complete the drawing in an $n \times m$ grid is $g = \text{g.c.d.}(n, m)$. Since all vertices of the drawing have degrees 2 or 4, Euler's theorem says there is a one-line drawing of that graph. An equivalent drawing is implemented by the Chokwe by placing mirror "walls" on some vertices so the curve bounces at those points, replacing an intersection with two curves, much like Bain's constructions of Celtic knots. A consequence of Euler's theorem is that the minimum number of such walls needed to create a one-line drawing is $g - 1$. But the artistic ideal of the Chokwe requires that the resulting drawing should be symmetric. It is always possible to construct symmetric sona using many such mirrors, but we ask when this is possible using the minimal number of such interior "walls". The result is that it is always possible for mirror symmetry, sometimes for rotational symmetry, and never for double mirror symmetry. The results include a variety of attractive, symmetric sona designs. (Received September 24, 2012)