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Ralph P. Grimaldi* (grimaldi@rose-hulman.edu), Professor Ralph P. Grimaldi, Mathematics Dept. RHIT, 5500 Wabash Avenue, Terre Haute, IN 47803. *Up-Down Sequences: Inversions, Coinversions, and the Sum of the Major Indices.*

For $n \geq 1$, let a_n count the number of up-down sequences s_1, s_2, \dots, s_n , where (i) $s_1 = 1$; (ii) $s_i \in \{1, 2, 3, 4\}$, for $2 \leq i \leq n$; and, (iii) $|s_i - s_{i-1}| = 1$, for $2 \leq i \leq n$.

One finds that $a_n = F_n$, the n -th Fibonacci number.

If s_1, s_2, \dots, s_n is a given up-down sequence, we say that (1) an inversion occurs whenever there are integers i, j where $1 \leq i < j \leq n$ but $s_i > s_j$; and, (2) a coinversion occurs whenever there are integers i, j where $1 \leq i < j \leq n$ and $s_i < s_j$.

We determine formulas for the numbers of inversions and coinversions that occur among these F_n sequences.

Finally, for an up-down sequence s of length n , we say that a descent occurs at position i , for $1 \leq i \leq n - 1$, if $s_i > s_{i+1}$, and we let $Des(s)$ be the set of all such positions. The major index of s is then the sum of all the entries in $Des(s)$. Our final result is a formula for the sum of the major indices for all F_n up-down sequences of length n . (Received September 18, 2012)