Shiyu Li* (jjl2357@berkeley.edu), Philip D. Tosteson (philip.d.tosteson@williams.edu) and Steven J. Miller (steven.j.miller@williams.edu). Distribution of the Longest Gap in Positive Linear Recurrence Sequences.

Throughout mathematics, and in number theoretic systems in particular, results on the spacing of objects have been key to understanding the objects themselves. A couple classic examples are the spacing between zeroes of the Dedekind zeta function, for distribution of primes in a number field, and the spectral gap between the largest two eigenvalues of the adjacency matrix of d-regular graphs.

Zeckendorf showed that every positive integer can be uniquely expressed as a sum of non-adjacent Fibonacci numbers. Beckwith and Miller further demonstrated that, in the limit, the distribution of gaps between summands is geometric. Using combinatorial techniques and probabilistic analysis, we extend this result and prove that the cumulative distribution of the longest gap is a doubly exponential function in the Fibonacci case, with critical value proportional to the logarithm of n for integers near the nth Fibonacci number. The probability of a longest gap rapidly tends to 0 or 1 as we move from this critical value. We then discover, based on this distribution function, a formula for the mean and variance for the spread of longest gaps. (Received September 25, 2012)