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**Gui-Qiang Chen\*** (gqchen@math.northwestern.edu), Department of Mathematics, Northwestern University, 2033 Sheridan Road, Evanston, IL 60208-2730, **Monica Torres** (torres@math.purdue.edu), Department of Mathematics, Purdue University, 150 N. University Street, West Lafayette, IN 47907-2067, and **William Ziemer** (ziemer@indiana.edu), Department of Mathematics, Indiana University, Rawles Hall, Bloomington, IN 47405. *Gauss-Green Theorem for Weakly Differentiable Vector Fields, Sets of Finite Perimeter, and Balance Laws.*

We analyze a class of weakly differentiable, bounded vector fields whose divergence is a Radon measure, i.e.  $\mathcal{DM}$ -fields. In particular, we establish a fundamental approximation theorem which states that, given a nonnegative Radon measure that is absolutely continuous with respect to  $\mathcal{H}^{N-1}$  on  $R^N$ , any set of finite perimeter can be approximated by a family of sets with smooth boundary essentially from the measure-theoretic interior of the set with respect to the measure. We then employ this approximation theorem to derive the normal trace of  $\mathcal{DM}$ -fields on the boundary of a set of finite perimeter, as the limit of the normal traces of  $\mathcal{DM}$ -fields on the boundaries of the approximate sets with smooth boundary, so that the Gauss-Green theorem holds on the set of finite perimeter. With these results, we analyze the Cauchy fluxes that are bounded by a Radon measure over any oriented surface and thereby develop a general mathematical formulation of the physical balance law through the Cauchy flux. Finally, we apply this framework to the derivation of systems of balance laws from the formulation and the recovery of Cauchy entropy fluxes through the Lax entropy inequality for entropy solutions of hyperbolic conservation laws. (Received September 03, 2007)