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Ziqin Feng* (zif1@pitt.edu), Department of Mathematics, Thackeray Hall 301, University of Pittsburgh, Pittsburgh, PA 15260. *Minimal Size of Basic Families.*

A family Φ of continuous real-valued functions on a space X is basic iff every map $f: X \rightarrow \mathbb{R}$ can be represented as a finite sum $f = (g_1 \circ \phi_1) + \dots + (g_n \circ \phi_n)$ for some ϕ_i in Φ and maps $g_i: \mathbb{R} \rightarrow \mathbb{R}$.

In his famous solution of Hilbert's 13th Problem, Kolmogorov proved that every compact metric finite dimensional space has a *finite* basic family.

Define a cardinal invariant of a space X by: $\text{basic}(X) = \min \{ |\Phi| : \Phi \text{ is a basic family for } X \}$.

Answering questions of Kolmogorov and others we have determined exactly which Tychonoff spaces have a basic finite family:

Theorem: The following are equivalent: i) X has $\text{basic}(X)$ finite, ii) X has $\text{basic}(X)$ countable, iii) X is locally compact, separable metrizable and finite dimensional.

More recently we have been calculating $\text{basic}(X)$ for compact spaces. The answers have intriguing connections with Shelah's PCF theory. (Received August 25, 2008)