1039-53-21Frederick Wilhelm* (fred@math.ucr.edu), Dept. of Mathematics, Univ. of Claf., Riverside,
CA 92521, and Ye-Lin Ou. Horizontally Homothetic Submersions and Nonnegative Curvature.

We study a generalization of Riemannian submersions called "horizontally homothetic" submersions.

Definition. A submersion of Riemannian manifolds, $\pi : M \to B$, is called horizontally homothetic if and only if there is a smooth function $\lambda : M \to \mathbb{R}$ with vertical gradient so that for all horizontal vectors x and y

$$\lambda^2 \langle x, y \rangle_M = \langle d\pi(x), d\pi(y) \rangle_B$$

 λ is called the dilation of π .

For this larger class of submersions, Gudmundsson and Kause and Washio showed that the analog of O'Neill's horizontal curvature equation has exactly one extra term. This extra term is always nonnegative and can potentially be positive. So the horizontal curvature equation suggests that a single horizontally homothetic submersion is more likely to have a positively curved image than a given Riemannian submersion. Since horizontally homothetic submersions are (a priori) more abundant, one is lead to believe that they have much more potential for creating positive curvature than Riemannian submersions. I will discuss the proof of a theorem that suggests that this is an illusion.

Theorem. Every horizontally homothetic submersion from a compact Riemannian manifold with nonnegative sectional curvature is a Riemannian submersion (up to a change of scale on the base space). (Received February 13, 2008)