

1036-49-21

Gilbert Strang* (gs@math.mit.edu), MIT Department of Mathematics, Cambridge, MA 02139.
Minimum Cuts and Maximum Area.

The oldest competition for an optimal shape (area-maximizing) was won by the circle. We propose a proof that uses the support function of the set—a dual description that linearizes the isoperimetric problem.

Then we measure the perimeter in different ways, which changes the problem (and has applications in medical imaging). If we use the line integral of $|dx| + |dy|$, a square would win. Or if the boundary integral of $\max(|dx|, |dy|)$ is given, a diamond has maximum area. When the perimeter $= \int \|(dx, dy)\|$ around the boundary is given, the area inside is maximized by a ball in the dual norm.

The second part describes the ****max flow-min cut theorem**** for continuous flows. Usually it is for discrete flows on edges of graphs. The maximum flow out of a region equals the capacity of the minimum cut. This duality connects to the constrained isoperimetric problems that produce minimum cuts, and to the Cheeger constant. (Received November 20, 2007)