$(3,6)$-fullerenes are cubic plane graphs in which all faces are hexagons, except for four faces that are triangles. There is a standard way of representing these graphs by folding a plane hexagonal lattice onto a suitable tetrahedron. Fowler and Cremona showed how to determine the automorphism group from this representation, but the conditions are complicated. By viewing the construction slightly differently we obtain simple conditions to determine the automorphism group and other structural information. We use this to address some conjectures of Déza and Dutour on "tight" graphs of this kind, and to describe the projective-planar cubic graphs with all faces hexagons except for two faces that are triangles. (Received February 01, 2009)

