1030-42-236

Ahmed I Zayed* (azayed@math.depaul.edu), Department of Mathematical Sciences, 2320 N. Kenmore Ave, Chicago, IL 60614, and Isaac Pesenson (pesenson@math.temple.edu). Paley-Wiener-Type Theorem in a Hilbert Space. Preliminary report.

The Paley-Wiener space $PW_{\sigma}(\mathbb{R})$ of bandlimited functions consists of entire functions of exponential type, with type $\leq \sigma$, that belong to $L^2(\mathbb{R})$ when restricted to \mathbb{R} . The Paley-Wiener theorem for bandlimited functions gives a nice characterization of the space $L^2[-\sigma,\sigma]$ under the Fourier transformation. A function $f \in L^2(\mathbb{R})$ belongs to $PW_{\sigma}(\mathbb{R})$ if and only if its L^2 -Fourier transform $\hat{f}(\omega)$ has support in $[-\sigma,\sigma]$.

Another characterization of the space $L^2[-\sigma,\sigma]$ that avoids complex variable techniques was given by Bang (Proc. Amer. Math. Soc. 1990): Let $1 \le p \le \infty$ and $f(x) \in C^{\infty}(\mathbb{R})$ such that $f^{(n)} \in L^p(\mathbb{R})$ for $n = 0, 1, \cdots$. Then the limit

$$\sigma_f = \lim_{n \to \infty} \left\| f^{(n)} \right\|_p^{1/n}$$

exists and

$$\sigma_f = \sup\left\{ |\omega| : \omega \in \operatorname{supp} \hat{f}(\omega) \right\},\$$

where \hat{f} is the Fourier transform of f. Such a characterization is called a Paley-Wiener-type characterization.

In this talk we introduce a Paley-Wiener-type theorem associated with a general self-adjoint operator in a Hilbert space. (Received August 03, 2007)