In this paper we show that a surface in $\mathbb{P}^{3}$ parametrized over a 2 -dimensional toric variety $T$ can be represented by a matrix of linear syzygies if the base points are finite in number and form locally a complete intersection. This constitutes a direct generalization of the corresponding result over $\mathbb{P}^{2}$ established by Busé, Chardin and Jouanolou. Exploiting the sparse structure of the parametrization, we obtain significantly smaller matrices than in the homogeneous case and the method becomes applicable to parametrizations for which it previously failed. We also treat the case $T=\mathbb{P}^{1} \times \mathbb{P}^{1}$ in detail and give numerous examples. (Received January 27, 2009)

