Gabor Pataki* (gabor@unc.edu), Dept of Statistics and Operations Research, Hanes Hall, UNC Chapel Hill, Chapel Hill, NC 27599, and Mustafa Tural (tural@email.unc.edu), Dept of Statistics and Operations Research, Hanes Hall, UNC Chapel Hill, Chapel Hill, NC 27599. Basis reduction, and the complexity of branch-and-bound.
Branch-and-bound is a classical method to solve integer programming feasibility problems. On the theoretical side, it is considered inefficient: it can provably take an exponential number of nodes to prove the infeasibility of a simple integer program.

In this work we show that branch-and-bound is theoretically efficient, if we apply a simple transformation in advance to the constraint matrix of the problem which makes the columns short and near orthogonal. We analyze two such reformulation methods, called the rangespace- and the nullspace methods. We prove that if the coefficients of the problem are drawn from $\{1, \ldots, M\}$ for a sufficiently large $M$, then for almost all such instances the number of subproblems that need to be enumerated by branch-and-bound is at most one.

Besides giving an analysis of branch-and-bound, our main result generalizes a result of Furst and Kannan on the solvability of subset sum problems to bounded integer programs.

We give some numerical values of $M$ which make sure that 99 percent of the reformulated problems solve at the rootnode. These values turned out to be surprisingly small for moderate values of $n$ (the number of variables), and $m$ (the number of constraints). (Received January 25, 2009)

