1049-90-68 **Bala Krishnamoorthy*** (`bkrishna@math.wsu.edu`), PO Box 643113 WSU, Pullman, WA 99164, and **William Webb** (`webb@math.wsu.edu`) and **Nathan Moyer** (`nmoyer@math.wsu.edu`).

Lattice-based Approaches to Number Partitioning in the Hard Phase.

The number partitioning problem (NPP) is to divide a set of integers $a_1, ..., a_n > 0$ into two disjoint subsets so that we minimize the difference of the subset sums, the discrepancy D. NPP is NP-complete, has a well-characterized phase transition, and finds many applications. When $a_j \in [1, R]$ (chosen uniformly), the expected optimal discrepancy is $O(\sqrt{n}2^{-n}R)$. The best known polynomial time approximation algorithm, due to Karmarkar and Karp, gives discrepancies that are $O(n^{-0.72 \log n}R)$. When $R > 2^n$, the optimal discrepancy is bigger than zero (no perfect partition), and the optimal partition is unique. Such instances with large R constitute the hard phase of NPP. We propose transformations of NPP in the hard phase to the shortest vector problem (SVP). We propose algorithms that tackle the NPP by solving SVP instances using basis reduction. We also propose a mixed integer program (MIP) model for NPP, and use a basis reduction-based reformulation of the MIP to handle the typically huge a_j's found in the hard phase. Finally, we propose a heuristic called truncated NPP, where we solve an equivalent NPP instance with $a'_j = a_j/T$ for $T > 0$. We show that the expected discrepancy given by this method is $O(D^* + nT)$, where D^* is the optimal discrepancy. (Received February 19, 2009)