For a positive integer $w$, let $Z^w$ denote the cartesian product of $w$ copies of the integers. When $A = (a_1, a_2, \ldots, a_w)$ and $B = (b_1, b_2, \ldots, b_w)$ are elements of $Z^w$, we say that $A$ and $B$ are $k$-crossing for some positive integer $k$ when there exist distinct integers $i$ and $j$ so that $a_i \geq k + b_i$ and $b_j \geq k + a_j$. Whenever $A$ and $B$ are incomparable, they are 1-crossing.

Now we have the following extremal problem: Find the maximum size $F(k, w)$ of an antichain $F$ in $Z^w$ so that no elements of $F$ are $k$-crossing. This problem was posed to us by Piotr Micek and he explained the bounds: $k^{w-1} \leq F(k, w) \leq k^w$. For $w = 1$ and $w = 2$, the lower bound is easily seen to be the correct answer, and it is conjectured that the lower bound is always correct. Bartosz Walczak has produced a very clever proof of this conjecture for the case $k = 3$.

In this talk, we discuss several constructions which explain why the lower bound is best possible and suggest lines of attack for the general case of the conjecture.

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