A rooted tree is \( k \)-ary if all non-leaves have \( k \) children; it is complete if all leaves have the same distance from the root. Let \( T \) be the complete ternary tree of depth \( n \). If each edge in \( T \) is labeled 0 or 1, then the labels along the edges of a path from the root to a leaf form a path label in \( \{0, 1\}^n \). Let \( f(n) \) be the maximum, over all \( \{0, 1\} \)-edge-labeled complete ternary trees \( T \) of depth \( n \), of the minimum number of distinct path labels on a complete binary subtree of depth \( n \) in \( T \).

The problem of bounding \( f(n) \) arose in studying a problem in computability theory, where it was hoped that \( f(n)/2^n \) tends to 0 as \( n \) grows. This is true; we show that \( f(n)/2^n \) is \( O(2^{-c\sqrt{n}}) \) for a positive constant \( c \). From below, we show that \( f(n) \geq (1.548)^n \) for sufficiently large \( n \). (Received January 19, 2011)