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**Rod Downey, Noam Greenberg, Carl G Jockusch and Kevin G Milans\***  
(milans@math.sc.edu). *Binary subtrees with few path labels.*

A rooted tree is *k-ary* if all non-leaves have  $k$  children; it is *complete* if all leaves have the same distance from the root. Let  $T$  be the complete ternary tree of depth  $n$ . If each edge in  $T$  is labeled 0 or 1, then the labels along the edges of a path from the root to a leaf form a *path label* in  $\{0, 1\}^n$ . Let  $f(n)$  be the maximum, over all  $\{0, 1\}$ -edge-labeled complete ternary trees  $T$  of depth  $n$ , of the minimum number of distinct path labels on a complete binary subtree of depth  $n$  in  $T$ .

The problem of bounding  $f(n)$  arose in studying a problem in computability theory, where it was hoped that  $f(n)/2^n$  tends to 0 as  $n$  grows. This is true; we show that  $f(n)/2^n$  is  $O(2^{-c\sqrt{n}})$  for a positive constant  $c$ . From below, we show that  $f(n) \geq (1.548)^n$  for sufficiently large  $n$ . (Received January 19, 2011)