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Stirling's formula offers ball-park estimates of $m!$. This three-century-old formula may be seen today as something that attracts amateur math enthusiasts, but it has actually got some interesting mathematics in it that has to do with the zeta function $\zeta(s)$. Numerous results that sharpen the Stirling's original estimates are known. The 'Euler-Maclaurin version' offers an intriguing link between $m!$ and the Bernoulli numbers, while it has one crucial drawback. Today I report on a different formula that has satisfactory features. It consists of under- and over-estimates of $\log m!$. The upshot of it is we can write both error terms as series of $(m+1)^{-1} \dots (m+r)^{-1}$. The under-estimate part is found in 'Wikipedia' (no mention of who discovered it), so our contribution is the over-estimate part. By truncating the series at an r -th term, we get refined (r -th degree) estimates of $m!$. I share with you 1. a new integral formula that expresses the Euler's constant $0.5772156\dots$, 2. a new functional identity of $\zeta(s)$, and 3. a new Wallis type product expression of the number $\zeta(3) = 1^{-3} + 2^{-3} + 3^{-3} + 4^{-3} + \dots$, all came out while studying the subject. (Received January 20, 2011)