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In [1] (J. London Math. Soc.,1995), a spectral reformulation of the Riemann hypothesis was obtained by M.L. Lapidus and H. Maier, involving inverse spectral problems for fractal strings. Later on, this work was revisited in light of the theory of complex dimensions of fractal strings developed by M.L. Lapidus and M. van Frankenhuysen in [2] (Fractal Geometry and Number Theory, Birkhauser,2000) and [3](Fractal Geometry,Complex Dimensions and Zeta Functions,Springer,2006). Moreover,in [3],the "spectral operator" was introduced as the operator that sends the geometry of a fractal string onto its spectrum. In the present work,we provide a rigorous functional analytic framework for the study of the spectral operator a . We show that a is an unbounded normal operator acting on a scale of Hilbert spaces (indexed by the Minkowski dimension D in $(0,1)$ of the underlying fractal strings), and precisely determine its spectrum. Furthermore, we deduce that for a given D , the spectral operator is invertible if and only if there are no Riemann zeros on the vertical line $\text{Re } s = D$. It follows that the associated inverse spectral problem has a positive answer for all possible dimensions D in $(0,1)$, other than in the mid-fractal case when $D = 1/2$, if and only if the Riemann hypothesis is true. (Received January 14, 2011)