The maximum subgraph problem for a fixed graph property P asks: Given a graph, find a subgraph satisfying property $P$ that has the maximum number of edges. This property can be planarity, acyclicity, bipartiteness, etc. We will discuss some old and new problems of this flavor with special emphasis on properties defined in terms of forbidden minors. In particular, we will describe some new algorithmic results on the maximum $K_{4}$ - minor-free subgraph problem (joint work with Calinescu and Fernandes). (Received March 30, 2010)

