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Yinghui Wang* (yinghui@mit.edu), 362 Memorial Dr, Cambridge, MA 02139, and **Steven J Miller**. *From Fibonacci numbers to Central Limit Type Theorems.*

Every integer is uniquely a sum of non-adjacent Fibonacci numbers $\{F_n\}$, and the average number of summands for integers in $[F_n, F_{n+1})$ is $n/(\varphi^2 + 1)$ with φ the golden mean. We prove the following massive generalization: for integers $c_1, \dots, c_L \geq 0$ with $c_1, c_L > 0$ and recursive sequence $\{H_n\}_{n=1}^\infty$ with $H_{n+1} = c_1 H_n + c_2 H_{n-1} + \dots + c_L H_{n+1-L}$ ($n \geq L$), $H_1 = 1$ and $H_{n+1} = c_1 H_n + c_2 H_{n-1} + \dots + c_n H_1 + 1$ ($1 \leq n < L$), every integer can be written uniquely as $\sum a_i H_i$ under natural constraints on the a_i 's, and the distribution of the number of summands converges to a Gaussian. Previous approaches were number theoretic, involving continued fractions, and were limited to results on existence and, in some cases, the mean. By recasting as a combinatorial problem and using generating functions and differentiating identities, we surmount these limitations.

Our method generalizes to many other problems. For example, every integer is uniquely a sum of the $\pm F_n$'s, such that every two terms of the same (opposite) sign differ in index by at least 4 (3). We prove the distribution of the numbers of positive and negative summands converges to a bivariate normal with correlation $-(21 - 2\varphi)/(29 + 2\varphi) \approx -0.551$. (Received January 26, 2011)