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Let R be a commutative ring with identity. A nonunit $a \in R$ is called an atom or said to be irreducible if whenever $a = bc$, $b, c \in R$, then $(a) = (b)$ or $(a) = (c)$ and R is said to be atomic if each nonunit of R is a finite product of atoms. R is called a (weak) Cohen-Kaplansky ring, or a (weak) CK ring for short, if R is atomic and (each maximal ideal of) R contains only finitely many nonassociate atoms. We show that the following conditions are equivalent: (1) R is a weak CK ring, (2) every (prime) ideal of R is a finite union of principal ideals, (3) R is atomic and every maximal ideal of R is a finite union of principal ideals, (4) R is a finite direct product of SPIRs, finite local rings and (one-dimensional) Noetherian domains in which every maximal ideal is a finite union of principal ideals, or equivalently, weak CK domains. The last equivalence effectively reduces the study of weak CK rings to weak CK domains. It is shown that an integral domain D is a weak CK domain if and only if D is Noetherian, for each maximal ideal M of D , D_M is a CK domain, and $\text{Pic}(D) = 0$. (Received November 30, 2010)