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Christopher Ernest. *Hyperbolic Euler Numbers and Polynomials.* Preliminary report.

Euler numbers are defined by

$$\frac{1}{\cosh(x)} = \frac{2e^x}{e^{2x} + 1} = \sum_{n=0}^{\infty} E_n \frac{x^n}{n!}.$$

We define a generalization of Euler numbers and polynomials by the generating functions

$$G_N(x) := \frac{2e^x}{e^{2x} + T_{N-1}(x)} = \sum_{n=0}^{\infty} E_{N,n} \frac{x^n}{n!},$$

and

$$G_N(x, z) := \frac{2e^{x(z+1)}}{e^{2x} + T_{N-1}(x)} = \sum_{n=0}^{\infty} E_{N,n}(z) \frac{x^n}{n!}$$

where

$$T_m(x) = \sum_{k=0}^m \frac{x^k}{k!}.$$

We refer these numbers and polynomials as *hyperbolic Euler numbers and polynomials of order N* . Note that $E_{1,n} = E_n$ and $E_{1,n}(z) = E_n(z)$ are the classical Euler numbers and polynomials, respectively. In this talk we will focus on $N = 2$ and consider some divisibility properties and give an explicit formula for these numbers. We will also prove similar result for the polynomials. For example, we will show that

$$E_{2,n} = 1 - \sum_{k=0}^{n-2} \binom{n}{k} 2^{n-k-1} E_{2,k}, - 2nE_{2,n-1}$$

and

$$E_{2,n} = n \sum_{k=0}^{n-1} \binom{n-1}{k} (-1)^{n-k} E_{2,k} - \sum_{k=0}^{n-1} \left(\frac{1 + (-1)^{n-k}}{2} \right) \binom{n}{k} E_{2,k}.$$

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