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Let V be a vector space of dimension m . For $1 \leq \ell_1 \leq \ell_2 < m$, consider

$$\mathcal{F}(\ell; m) = \{(V_1, V_2) \in G_{\ell_1}(V) \times G_{\ell_2}(V) : V_1 \subseteq V_2\}$$

with its Plücker-Segre embedding in $\mathbb{P}(\wedge^{\ell_1} V \otimes \wedge^{\ell_2} V)$. We consider the following questions:

- Determine the least positive integer $k \leq \binom{m}{\ell_1} \binom{m}{\ell_2}$ such that $\mathcal{F}(\ell; m)$ embeds in \mathbb{P}^{k-1} .
- Determine the maximal number of \mathbb{F}_q -rational points in $\mathcal{F}(\ell; m) \cap H$, where H varies over hyperplanes in \mathbb{P}^{k-1} .

Questions such as these have been considered by several mathematicians in the case of Veroneseans and Grassmannians. For the line-hyperplane incidence variety $\mathcal{F}(\ell; m)$ where $\ell = (1, m - 1)$, both these questions were answered by F. Rodier (2003). Subsequently, there has been some partial progress by G. Hana (2005), but the general case appears to be open. We answer the first question using some combinatorial representation theory. We also give an explicit lower bound, and in some cases the exact value, for the maximum sought by the second question. (Received September 20, 2016)