1125-VF-1377   **Michael D. Barrus*** (`barrus@uri.edu`) and **John Sinkovic**. *Uniqueness in labelings of tree-depth-critical graphs.*

A k-ranking of a graph G is a labeling of the vertices of G with values from $\{1, ..., k\}$ such that any path joining two vertices with the same label contains a vertex having a higher label. The tree-depth of G is the smallest value of k for which a k-ranking of G exists. The graph G is critical if every proper minor of G has smaller tree-depth than G has.

Focusing on k-rankings of critical graphs, we define a graph G to be 1-unique if for every vertex v in G, there exists an optimal ranking in which v is the unique vertex with label 1. We explore the seemingly close relationships between 1-uniqueness and criticality, showing that (contrary to an earlier conjecture) not all critical graphs are 1-unique, though many are. We show that for graphs that are 1-unique, we have useful tools to more easily conclude criticality, construct larger critical graphs, and prove conjectured properties of critical graphs. (Received September 16, 2016)